M11/5/MATME/SP2/ENG/TZ2/XX/M

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MARKSCHEME

May 2011

MATHEMATICS

Standard Level

Paper 2

17 pages

Instructions to Examiners

Abbreviations

M Marks awarded for attempting to use a correct **Method**; working must be seen.

(M) Marks awarded for **Method**; may be implied by **correct** subsequent working.

A Marks awarded for an **Answer** or for **Accuracy**; often dependent on preceding ***M*** marks.

(A) Marks awarded for an **Answer** or for **Accuracy**; may be implied by **correct** subsequent working.

R Marks awarded for clear **Reasoning**.

N Marks awarded for **correct** answers if **no** working shown.

AG Answer given in the question and so no marks are awarded.

Using the markscheme

1 General

Mark according to scoris instructions and the document "**Mathematics SL : Guidance for e-marking May 2011**". It is **essential** that you read this document before you start marking. In particular, please note the following.

Marks must be recorded using the annotation stamps. Please check that you are entering marks for the right question.

- If a part is **completely correct**, (and gains all the 'must be seen' marks), use the ticks with numbers to stamp full marks.
- If a part is completely wrong, stamp ***A0*** by the final answer.
- If a part gains anything else, it **must** be recorded using **all** the annotations.

All the marks will be added and recorded by scoris.

2 Method and Answer/Accuracy marks

- Do **not** automatically award full marks for a correct answer; all working **must** be checked, and marks awarded according to the markscheme.
- It is generally not possible to award ***M0*** followed by ***A1***, as ***A*** mark(s) depend on the preceding ***M*** mark(s), if any. An exception to this rule is when work for ***M1*** is missing, as opposed to incorrect (see point 4).
- Where ***M*** and ***A*** marks are noted on the same line, *e.g.* ***M1A1***, this usually means ***M1*** for an **attempt** to use an appropriate method (*e.g.* substitution into a formula) and ***A1*** for using the **correct** values.
- Where there are two or more ***A*** marks on the same line, they may be awarded independently; so if the first value is incorrect, but the next two are correct, award ***A0A1A1***.
- Where the markscheme specifies ***(M2)***, ***N3***, *etc*., do **not** split the marks, unless there is a note. Once a correct answer to a question or part-question is seen, ignore further working.
- ***M*** marks are generally awarded for a valid attempt at answering the question. The working here does not need to be correct. In particular, where this is not the first ***M*** mark awarded, the markscheme may use the word "their", to indicate that even if there are previous errors, the ***M*** mark may still be awarded. A common example of this is where candidates need to derive an equation and then the markscheme says "attempt to solve **their** equation"; if their equation is wrong and as long as it is not simpler than the correct one, then the ***M*** mark is awarded.

3 *N* marks

*If **no** working shown, award **N** marks for **correct** answers. In this case, ignore mark breakdown (**M, A, R**).*

- Do **not** award a mixture of ***N*** and other marks.
- There may be fewer ***N*** marks available than the total of ***M***, ***A*** and ***R*** marks; this is deliberate as it penalizes candidates for not following the instruction to show their working.
- There may not be a direct relationship between the ***N*** marks and the implied marks. There are times when all the marks are implied, but the ***N*** marks are not the full marks: this indicates that we want to see some of the working, without specifying what.
- For consistency within the markscheme, ***N*** marks are noted for every part, even when these match the mark breakdown.
- If a candidate has incorrect working, which somehow results in a correct answer, do **not** award the ***N*** marks for this correct answer. However, if the candidate has indicated (usually by crossing out) that the working is to be ignored, award the ***N*** marks for the correct answer.

4 Implied and must be seen marks

*Implied marks appear in **brackets e.g. (M1)**.*

- Implied marks can only be awarded if **correct/appropriate** work is seen or implied in subsequent working. There are questions where some working is required, but as it is accepted that not everyone will write the same steps, all the marks are implied, but the ***N*** marks are not the full marks for the question. (A correct final answer does not necessarily mean that the implied marks are all awarded).
- Normally the work is seen or implied in the next line.
- Where there is an ***(M1)*** followed by ***A1*** for each correct value, if no working shown, one correct value is sufficient evidence to award the ***(M1)***.

*Must be seen marks appear without **brackets e.g. M1**.*

- Must be seen marks can only be awarded if **correct/appropriate** work is seen.
- If a must be seen mark is not awarded because work is missing (as opposed to ***M0*** or ***A0*** for incorrect work) all subsequent marks may be awarded if appropriate.

5 Follow through marks (only applied after an error is made)

*Follow through (**FT**) marks are awarded where an incorrect answer from one **part** of a question is used correctly in **subsequent** part(s) or subpart(s). Usually, to award **FT** marks, **there must be working present** and not just a final answer based on an incorrect answer to a previous part. However, if the only marks awarded in a subpart are for the answer (i.e. there is no working expected), then **FT** marks should be awarded if appropriate.*

- Within a question part, once an **error** is made, no further ***A*** marks can be awarded for work which uses the error, but ***M*** marks may be awarded if appropriate. (However, as noted above, if an ***A*** mark is not awarded because work is missing, all subsequent marks may be awarded if appropriate).
- If the question becomes much simpler because of an error then use discretion to award fewer ***FT*** marks.
- If the error leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Exceptions to this rule will be explicitly noted on the markscheme.
- If a candidate makes an error in one part, but gets the correct answer(s) to subsequent part(s), award marks as appropriate, unless the question says hence. It is often possible to use a different approach in subsequent parts that does not depend on the answer to previous parts.
- ***FT*** and "show that" questions : In a "show that" question, if an error leads to not showing the required answer, award ***FT*** marks as usual but do not award the final ***A1***.

6 Mis-read

*If a candidate incorrectly copies information from the question, this is a mis-read (**MR**). Apply a **MR** penalty of 1 mark to that question. Award the marks as usual and then stamp **MR** against the answer. Scoris will automatically deduct 1 mark from the question total. A candidate should be penalized only once for a particular mis-read. Do not stamp **MR** again for that question, unless the candidate makes another mis-read.*

- If the question becomes much simpler because of the ***MR***, then use discretion to award fewer marks.
- If the ***MR*** leads to an inappropriate value (*e.g.* probability greater than 1, use of $r > 1$ for the sum of an infinite GP, $\sin\theta = 1.5$, non integer value where integer required), do not award the mark(s) for the final answer(s).
- Miscopying of candidates' own work does **not** constitute a misread, it is an error.

7 Discretionary marks *(d)*

*An examiner uses discretion to award a mark on the rare occasions when the markscheme does not cover the work seen. The mark should be labelled **(d)** and a brief **note** written next to the mark explaining this decision.*

8 Alternative methods

Candidates will sometimes use methods other than those in the markscheme. Unless the question specifies a method, other correct methods should be marked in line with the markscheme. If in doubt, contact your team leader for advice.

- Alternative methods for complete questions are indicated by **METHOD 1**, **METHOD 2**, *etc*.
- Alternative solutions for part-questions are indicated by **EITHER** . . . **OR**.
- Where possible, alignment will also be used to assist examiners in identifying where these alternatives start and finish.

9 Alternative forms

*Unless the question specifies otherwise, **accept** equivalent forms.*

- As this is an international examination, accept all alternative forms of **notation**.
- In the markscheme, equivalent **numerical** and **algebraic** forms will generally be written in brackets immediately following the answer.
- In the markscheme, **simplified** answers, (which candidates often do not write in examinations), will generally appear in brackets. Marks should be awarded for either the form preceding the bracket or the form in brackets (if it is seen).

10 Accuracy of Answers

If the level of accuracy is specified in the question, a mark will be allocated for giving the answer to the required accuracy.

- **Rounding errors**: only applies to final answers not to intermediate steps.
- **Level of accuracy**: when this is not specified in the question the general rule applies to final answers: *unless otherwise stated in the question all numerical answers must be given exactly or correct to three significant figures*.

*Candidates should be penalized **once only IN THE PAPER** for an accuracy error **(AP)**. Award the marks as usual then stamp **(AP)** against the answer. Scoris will automatically deduct 1 mark from the paper total.* Please see section E in the guidance document which clearly explains, with the use of screenshots, how this works in scoris.

- If a final correct answer is incorrectly rounded, apply the ***AP***.
- If the level of accuracy is not specified in the question, apply the ***AP*** for correct final answers not given to three significant figures.
- Intermediate values are sometimes written as 3.24(741). This indicates that using 3.24 (or 3.25) is acceptable, but the more accurate value is 3.24741. The digits in brackets are not required for the marks. If candidates work with fewer than three significant figures, this could lead to an ***AP***.
- Do not accept unfinished numerical answers such as 3/0.1 (unless otherwise stated). As a rule, numerical answers with more than one part (such as fractions) should be given using integers (*e.g.* 6/8).

If there is no working shown, and there is an accuracy error (*i.e.* answers are given to the correct two significant figures, or incorrectly rounded but given to three significant figures), apply the ***AP*** with the ***N*** marks for these answers. However, do **not** accept answers to one significant figure without working.

11 Calculators

A GDC is required for paper 2, but calculators with symbolic manipulation features (e.g. TI-89) are not allowed.

Calculator notation
The Mathematics SL guide says:

Students must always use correct mathematical notation, not calculator notation.

Do **not** accept final answers written using calculator notation. However, do not penalize the use of calculator notation in the working.

12 Style

The markscheme aims to present answers using good communication, e.g. if the question asks to find the value of k, the markscheme will say $k = 3$*, but the marks will be for the correct value 3 – there is usually no need for the "* $k =$ *". In these cases, it is also usually acceptable to have another variable, as long as there is no ambiguity in the question, e.g. if the question asks to find the value of p and of q, then the student answer needs to be clear. Generally, the only situation where the full answer is required is in a question which asks for equations – in this case the markscheme will say "must be an equation".*

The markscheme often uses words to describe what the marks are for, followed by examples, using the e.g. notation. These examples are not exhaustive, and examiners should check what candidates have written, to see if they satisfy the description. Where these marks are ***M*** *marks, the examples may include ones using poor notation or incorrect work, to indicate what is acceptable.*

13 Crossed out work

If a candidate has drawn a line through work on their examination script, or in some other way crossed out their work, do not award any marks for that work.

14 More than one solution

Where a candidate offers two or more different answers to the same question, the candidate knows one of the answers must be wrong or relatively inferior. This is likely to be an effort by the candidate to gain an advantage, hoping that the examiner will give credit for the right answer while the wrong or inferior answer is ignored. **In such circumstances an examiner should only mark the first response unless the candidate indicates otherwise**.

SECTION A

1. (a) attempt to form composite *(M1)*
e.g. $f(2x-5)$

$h(x)=6x-15$ *A1* *N2*

[2 marks]

(b) interchanging x and y *(M1)*

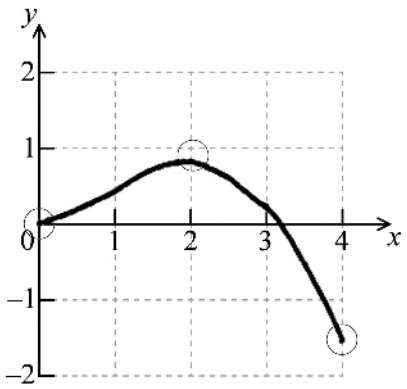
evidence of correct manipulation *(A1)*
e.g. $y+15=6x$, $\frac{x}{6}=y-\frac{5}{2}$

$h^{-1}(x)=\frac{x+15}{6}$ *A1* *N3*

[3 marks]

Total [5 marks]

2. (a)



A1A1A1A1 *N4*

Note: Award ***A1*** for approximately correct shape, ***A1*** for left end point in circle, ***A1*** for local maximum in circle, ***A1*** for right end point in circle.

[4 marks]

(b) attempting to solve $g(x)=-1$ *(M1)*

e.g. marking coordinate on graph, $\frac{1}{2}x\sin x+1=0$

$x=3.71$ *A1* *N2*

[2 marks]

Total [6 marks]

3. (a) 12 terms *A1* *N1*
[1 mark]

(b) evidence of binomial expansion *(M1)*
e.g. $\binom{n}{r}a^{n-r}b^r$, an attempt to expand, Pascal's triangle

evidence of choosing correct term *(A1)*
e.g. 10^{th} term, $r=9$, $\binom{11}{9}$, $(x)^2(2)^9$

correct working *A1*
e.g. $\binom{11}{9}(x)^2(2)^9$, 55×2^9

$28160x^2$ *A1* *N2*
[4 marks]

Total [5 marks]

4. (a) $\boldsymbol{M}=\begin{pmatrix}1 & 6 & -3\\ 4 & 2 & -4\\ 1 & 1 & 5\end{pmatrix}$, $\boldsymbol{N}=\begin{pmatrix}-1\\ 12\\ 15\end{pmatrix}$ *A2A1* *N3*
[3 marks]

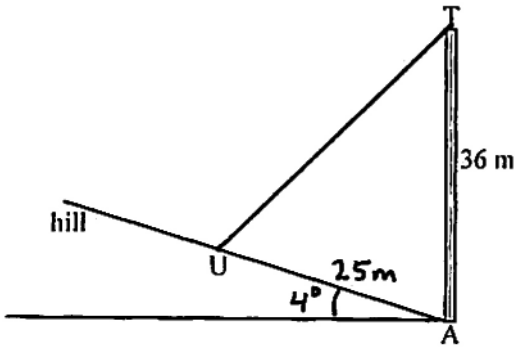
(b) evidence of appropriate approach *(M2)*
e.g. $\boldsymbol{X}=\boldsymbol{M}^{-1}\boldsymbol{N}$, attempting to solve a system of three equations

$\boldsymbol{X}=\begin{pmatrix}5\\ 0\\ 2\end{pmatrix}$ *A1* *N3*
[3 marks]

(c) $x=5$, $y=0$, $z=2$ *A1* *N1*
[1 mark]

Total [7 marks]

5. (a)



A1A1A1 ***N3***

Note: Award ***A1*** for labelling 4° with horizontal, ***A1*** for labelling [AU] 25 metres, ***A1*** for drawing [TU].

[3 marks]

(b) $\hat{TAU} = 86^\circ$ ***(A1)***

evidence of choosing cosine rule ***(M1)***

correct substitution ***A1***
e.g. $x^2 = 25^2 + 36^2 - 2(25)(36)\cos 86^\circ$

$x = 42.4$ ***A1*** ***N3***

[4 marks]

Total [7 marks]

6. (a) symmetry of normal curve ***(M1)***
e.g. $\mathrm{P}(X < 25) = 0.5$

$\mathrm{P}(X > 27) = 0.2$ ***A1*** ***N2***
[2 marks]

(b) **METHOD 1**

finding standardized value ***(A1)***
e.g. $\frac{27-25}{\sigma}$

evidence of complement ***(M1)***
e.g. $1-p$, $\mathrm{P}(X < 27)$, 0.8

finding *z*-score ***(A1)***
e.g. $z = 0.84\ldots$

attempt to set up equation involving the standardized value ***M1***
e.g. $0.84 = \frac{27-25}{\sigma}$, $0.84 = \frac{X-\mu}{\sigma}$

$\sigma = 2.38$ ***A1*** ***N3***
[5 marks]

METHOD 2

set up using normal CDF function and probability ***(M1)***
e.g. $\mathrm{P}(25 < X < 27) = 0.3$, $\mathrm{P}(X < 27) = 0.8$

correct equation ***A2***
e.g. $\mathrm{P}(25 < X < 27) = 0.3$, $\mathrm{P}(X > 27) = 0.2$

attempt to solve the equation using GDC ***(M1)***
e.g. solver, graph, trial and error (more than two trials must be shown)

$\sigma = 2.38$ ***A1*** ***N3***
[5 marks]

Total [7 marks]

7. METHOD 1

evidence of antidifferentiation ***(M1)***

e.g. $\int (10e^{2x} - 5)\,dx$

$y = 5e^{2x} - 5x + C$ ***A2A1***

Note: Award ***A2*** for $5e^{2x}$, ***A1*** for $-5x$. If "C" is omitted, award no further marks.

substituting $(0, 8)$ ***(M1)***

e.g. $8 = 5 + C$

$C = 3$ $(y = 5e^{2x} - 5x + 3)$ ***(A1)***

substituting $x = 1$ ***(M1)***

$y = 34.9$ $(5e^2 - 2)$ ***A1*** ***N4***

[8 marks]

METHOD 2

evidence of definite integral function expression ***(M2)***

e.g. $\int_a^x f'(t)\,dt = f(x) - f(a)$, $\int_0^x (10e^{2x} - 5)$

initial condition in definite integral function expression ***(A2)***

e.g. $\int_0^x (10e^{2t} - 5)\,dt = y - 8$, $\int_0^x (10e^{2x} - 5)\,dx + 8$

correct definite integral expression for y when $x = 1$ ***(A2)***

e.g. $\int_0^1 (10e^{2x} - 5)\,dx + 8$

$y = 34.9$ $(5e^2 - 2)$ ***A2*** ***N4***

[8 marks]

SECTION B

8. (a) appropriate approach ***(M1)***

e.g. $\vec{AO}+\vec{OB}$, $B-A$

$\vec{AB}=\begin{pmatrix}1\\-1\\1\end{pmatrix}$ ***A1*** ***N2***

[2 marks]

(b) **any** correct equation in the form $\boldsymbol{r}=\boldsymbol{a}+t\boldsymbol{b}$ ***A2*** ***N2***

where $\boldsymbol{b}$ is a scalar multiple of $\begin{pmatrix}1\\-1\\1\end{pmatrix}$

e.g. $\boldsymbol{r}=\begin{pmatrix}1\\-1\\4\end{pmatrix}+t\begin{pmatrix}1\\-1\\1\end{pmatrix}$, $\boldsymbol{r}=\begin{pmatrix}2+t\\-2-t\\5+t\end{pmatrix}$, $\boldsymbol{r}=2\boldsymbol{i}-2\boldsymbol{j}+5\boldsymbol{k}+t(\boldsymbol{i}-\boldsymbol{j}+\boldsymbol{k})$

[2 marks]

(c) choosing correct direction vectors $\begin{pmatrix}2\\1\\3\end{pmatrix}$, $\begin{pmatrix}1\\-1\\1\end{pmatrix}$ ***(A1)(A1)***

finding scalar product and magnitudes ***(A1)(A1)(A1)***

scalar product $=1\times2+-1\times1+1\times3\quad(=4)$

magnitudes $\sqrt{1^2+(-1)^2+1^2}\ (=1.73\ldots)$, $\sqrt{4+1+9}\ (=3.74\ldots)$

substitution into $\frac{\boldsymbol{u}\bullet\boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$ $\left(\text{accept } \theta=\frac{\boldsymbol{u}\bullet\boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}, \text{ but not } \sin\theta=\frac{\boldsymbol{u}\bullet\boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}\right)$ ***M1***

e.g. $\cos\theta=\frac{1\times2+-1\times1+1\times3}{\sqrt{1^2+(-1)^2+1^2}\sqrt{2^2+1^2+3^2}}$, $\cos\theta=\frac{4}{\sqrt{42}}$

$\theta=0.906\ \ (51.9^\circ)$ ***A1*** ***N5***

[7 marks]

continued ...

Question 8 continued

(d) **METHOD 1** $\left(\text{from } \boldsymbol{r} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right)$

appropriate approach ***(M1)***

e.g. $\boldsymbol{p} = \boldsymbol{r}$, $\begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} + \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} t = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} s$,

two **correct** equations ***A1A1***

e.g. $1 + t = 2 + 2s$, $-1 - t = 4 + s$, $4 + t = 7 + 3s$

attempt to solve ***(M1)***

one correct parameter ***A1***

e.g. $t = -3$, $s = -2$

C is $(-2, 2, 1)$ ***A1*** ***N3***

[6 marks]

METHOD 2 $\left(\text{from } \boldsymbol{r} = \begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} \right)$

appropriate approach ***(M1)***

e.g. $\boldsymbol{p} = \boldsymbol{r}$, $\begin{pmatrix} 2 \\ -2 \\ 5 \end{pmatrix} + t \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 4 \\ 7 \end{pmatrix} + \begin{pmatrix} 2 \\ 1 \\ 3 \end{pmatrix} s$,

two **correct** equations ***A1A1***

e.g. $2 + t = 2 + 2s$, $-2 - t = 4 + s$, $5 + t = 7 + 3s$

attempt to solve ***(M1)***

one correct parameter ***A1***

e.g. $t = -4$, $s = -2$

C is $(-2, 2, 1)$ ***A1*** ***N3***

[6 marks]

Total [17 marks]

9. (a) three correct pairs ***A1A1A1*** ***N3***
e.g. $(2, 4), (3, 3), (4, 2)$, $R2G4$, $R3G3$, $R4G2$

[3 marks]

(b) $p = \frac{1}{16}$, $q = \frac{2}{16}$, $r = \frac{2}{16}$ ***A1A1A1*** ***N3***

[3 marks]

(c) let X be the number of times the sum of the dice is 5

evidence of valid approach ***(M1)***
e.g. $X \sim \mathrm{B}(n, p)$, tree diagram, 5 sets of outcomes produce a win

one correct parameter ***(A1)***
e.g. $n = 4$, $p = 0.25$, $q = 0.75$

Fred wins prize is $\mathrm{P}(X \geq 3)$ ***(A1)***

appropriate approach to find probability ***M1***
e.g. complement, summing probabilities, using a CDF function

correct substitution ***(A1)***
e.g. $1 - 0.949\ldots$, $1 - \frac{243}{256}$, $0.046875 + 0.00390625$, $\frac{12}{256} + \frac{1}{256}$

probability of winning $= 0.0508$ $\left(\frac{13}{256}\right)$ ***A1*** ***N3***

[6 marks]

Total [12 marks]

10. (a) evidence of finding height, h ***(A1)***

e.g. $\sin\theta = \frac{h}{2}$, $2\sin\theta$

evidence of finding base of triangle, b ***(A1)***

e.g. $\cos\theta = \frac{b}{2}$, $2\cos\theta$

attempt to substitute valid values into a formula for the area of the window ***(M1)***
e.g. two triangles plus rectangle, trapezium area formula

correct expression (must be in terms of θ) ***A1***

e.g. $2\left(\frac{1}{2}\times 2\cos\theta\times 2\sin\theta\right)+2\times 2\sin\theta$, $\frac{1}{2}(2\sin\theta)(2+2+4\cos\theta)$

attempt to replace $2\sin\theta\cos\theta$ by $\sin 2\theta$ ***M1***

e.g. $4\sin\theta + 2(2\sin\theta\cos\theta)$

$y = 4\sin\theta + 2\sin 2\theta$ ***AG*** ***N0***

[5 marks]

(b) correct equation ***A1***

e.g. $y = 5$, $4\sin\theta + 2\sin 2\theta = 5$

evidence of attempt to solve ***(M1)***

e.g. a sketch, $4\sin\theta + 2\sin\theta - 5 = 0$

$\theta = 0.856\ (49.0^\circ)$, $\theta = 1.25\ (71.4^\circ)$ ***A1A1*** ***N3***

[4 marks]

(c) recognition that lower area value occurs at $\theta = \frac{\pi}{2}$ ***(M1)***

finding value of area at $\theta = \frac{\pi}{2}$ ***(M1)***

e.g. $4\sin\left(\frac{\pi}{2}\right)+2\sin\left(2\times\frac{\pi}{2}\right)$, draw square

$A = 4$ ***(A1)***

recognition that maximum value of y is needed ***(M1)***

$A = 5.19615\ldots$ ***(A1)***

$4 < A < 5.20$ (accept $4 < A < 5.19$) ***A2*** ***N5***

[7 marks]

Total [16 marks]